On the Ground Response Curve

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In NATM-tunnelling the ground response curve, also referred to as Fenner-Pacher curve, is used to illustrate the ground pressure on the lining as a function of deformations. A steep ground response curve with a low minimum indicates a stiff and strong ground which needs little support of a lining. Such a cohesive and frictional soil is able to carry the overburden load by arching around the tunnel. Vice versa, a relatively flat ground response curve with a high minimum corresponds to a relatively soft ground that needs significant support from a lining. The hypothesis by Pacher (10) concerning the trough-shaped ground response curve and the minimisation of the rock pressure and the related lining thickness would seem to be important elements of NATM-tunnelling. The idea of a trough-shaped ground response curve was however questioned by Kovári (7) and thereupon defended by Vavrovsky (15). The latter argues that distinction should be made between shallow and deep tunnelling. On the basis of his tunnelling experiences, Vavrovsky considered a concave ground response curve as realistic for shallow tunnels rather than for deep tunnels. In this study these ideas will be confirmed on the basis of numerical analyses.

Attention will be focused on relatively shallow tunnels in isotropic ground without a macro structure due to stratification, schistarity or jointing. Material behaviour will be described by an elastoplastic constitutive model that involves softening. This model will be used in finite element analyses in order to compute ground response curves. The idea of calculating ground response curves on the basis of elementary ground properties was probably first suggested by Seeber (13) and later by Kovári (6) and Kolymbas (5). Most recently this was done for deep tunnels by Bliem and Fellin (2) to find non-concave curves. In contrast, we will consider shallow tunnels to find trough-like ground response curves.

Finite element-analyses in softening ground

Several authors (16, 17) have shown that the elastic-plastic finite-element method is well-suited to predict collapse loads of geotechnical structures. For softening ground, however, such

Untersuchungen zur Gebirgskennlinie

Im vorliegenden Beitrag wird das mechanische Verhalten von Tunnels in steifen Böden und „weichem“, nachgiebigem Fels numerisch untersucht. Der Schwerpunkt liegt bei der Analyse von seicht bis mitteltief liegenden NÖT-Tunneln. Es wird gezeigt, daß sich in diesen Fällen im Unterschied zu sehr tiefliegenden Tunnel durch den progressiven Entfestigungsprozeß ein Bruch ergeben kann, der durch die sogenannte ansteigende Gebirgskennlinie veranschaulicht wird.

This numerical study concerns the mechanical behaviour of tunnels in stiff soils and soft rocks. Attention is focused on shallow to medium deep NATM tunnels. It is shown that such situations differ from very deep tunnels in the sense that material softening can produce failure, as demonstrated by a trough-like ground response curve.

numerical analyses tend to require a considerable computational effort. At present such computations are feasible for two-dimensional problems rather than for three-dimensional ones. In the present study such a two-dimensional problem is considered. The finite element analyses were performed with an advanced constitutive model that accounts for a drop in strength, i.e. softening. This model will be briefly introduced in a separate section.

As symmetrical tunnels are considered, calculations are based on only half a circular tunnel. The ground is represented by 6-noded triangular elements. The boundary conditions of the finite element mesh are as follows: The ground surface is free to displace, the side surfaces have roller boundaries and the base is fixed. It is assumed that the distribution of the initial stresses is geostatic according to $\sigma'_h = K_0\, \sigma'_v$, where σ'_h is the horizontal effective stress and σ'_v is the vertical one. K_0 is the coefficient of lateral earth pressure at rest as illustrated in Figure 1.

The first stage of the calculations is to remove the elements inside the tunnel. This does not disturb the equilibrium as equivalent pressures are applied on the inside of the entire tunnel. The minimum amount of pressure needed to support the tunnel is then determined by a stepwise reduction of the supporting pressure.

Upon extending finite element procedures to include softening, it appears that the entire numerical procedure should be well designed in order that an accurate assessment of the ground response can be made. For each decrement of supporting pressure, equilibrium iterations are performed and plastic stress redistribution is accomplished using a radial-return algorithm in combination with so-called arc-length control (3). In this manner ground response curves, being also known as Fenner-Pacher curves, are obtained for an assumed plane strain tunnelling situation. In the following such curves will be obtained by plotting the average supporting pressure as a function of the roof settlement.

Within the context of classical continuum mechanics and conventional finite element analyses, softening models create mesh dependency. Hence, computational results will depend on the size of the elements being used. In order to obtain objective results that are independent of the finite element mesh, classical continuum models have to be enhanced by a so-called regularization technique. In this study the nonlocal method (1) is used to achieve mesh independent computational results. Within this method the shear band thickness is governed by an internal length parameter. Unfortunately this additional input parameter is difficult to measure and this renders the use of softening analyses presently non-attractive for use in engineering practise. For details on this parameter as well as the so-called nonlocal method being used, the reader is referred to Marcher (9).

Fig. 1 Geometry and typical pressure-displacement curve (ground response curve).

Bild 1 *Tunnelgeometrie und eine typische Druck-Verschiebungskurve (Gebirgskennlinie).*

Constitutive model with cohesion softening

Stiff clays and weak rocks tend to show a peak strength and a much lower residual strength. The transition from the peak to the residual strength is referred to as softening and it usual occurs in combination with the localisation of deformations in shear bands, i.e. thin zones of intensively shearing material. Softening shear bands imply a reduction of shear stresses both inside and outside the band; otherwise there would be no equilibrium. In adjacent regions outside the band the shear stress reduction causes a quasi-elastic unloading so that one observes more or less rigid block movements. The resulting tendency of progressive failure is well-known for clay slopes (4, 11, 14). In tunnelling, softening may result in a concave ground-response curve as considered e.g. by Pacher (10) and more recently by Vavrovsky (15).

In stiff clays softening occurs both for the friction angle and the cohesion, but friction softening is not as dangerous as cohesion softening. This relates to the fact that bonds between particles conferring effective cohesion are destroyed after small deformations, finally resulting in zero cohesion. In contrast, friction angles tend to drop much more slowly down to a residual value well above zero. A gradual loss of stability due to ductile friction softening is thus less severe and more readily observed within the framework of a monitoring programme, whereas more brittle cohesion softening may lead to a more sudden loss of stability. It is thus logic to concentrate on cohesion softening.

Instead of the use of a simple elastic-perfectly plastic model such as the Mohr-Coulomb model, a hardening-softening model is adopted. This is basically the so-called Hardening-Soil Model as used in the Plaxis program (3), but this constitutive model was extended by adding softening behaviour. Rather than describing the mathematical formulation of this particular constitutive soil model, full focus will be on the meaning of the input parameters of this model. The considered ground has a unit weight of $\gamma = 20$ kN/m³, a friction angle of $\varphi' = 30°$ and an initial effective cohesion of $c_{peak} = 40$ kPa. Similar to the strength parameters, stiffness parameters are taken conform the properties of the stiff clay tested at Stuttgart University (8).

Cohesion softening is modelled by considering the effective cohesion to be a function of the void ratio e. On introducing a constant softening modulus h_c, the cohesion is written as

$$c' = c_{peak} - h_c\,(e - e_o) \qquad [1]$$

as illustrated in figure 2. The constants c_{peak} and e_0 are initial values at the onset of tunnelling. For details on this constitutive model, the reader is referred to Marcher (9). The softening modulus

Fig. 2 a) Typical load displacement curve for a triaxial test on softening material; b) linear cohesion softening.

Bild 2 *a) Typische Last-Verschiebungskurve eines Triaxialversuchs an einem entfestigenden Material; b) lineare Kohäsionsentfestigung.*

h_c can be obtained on the basis of high-quality triaxial tests with a relatively homogeneous post-peak sample deformations. At Stuttgart University such tests were carried out on a particular stiff clay, named Beaucaire Marl (8), to find h_c = 600 kPa. This particular value is to be used as a reference value when studying the effect of softening in tunnel stability. No doubt, different clays will give different values of h_c, but the number of 600 kPa can at least be used as a reference value in the context of a sensitivity analysis. The experiments on Beaucaire Marl showed cohesion softening rather than friction softening.

Stiffness parameters being used

Figure 3 shows a typical curve of a drained triaxial test with constant lateral pressure σ_3. Under primary loading the behaviour is distinctly non-linear and is assumed to be hyperbolic up to a failure stress. Here compressive stresses and strains are considered positive. While the maximum stress is determined by the Mohr-Coulomb failure criterion, the hyperbolic part of the curve can be defined by using a single secant modulus as additional input parameter. In the hardening-softening model this is E_{50}, as shown in Figure 3. It determines the magnitude of both the elastic and the plastic strains. In contrast, E_{ur} is an elasticity modulus. In conjunction with a Poisson's ratio ν_{ur}, the elasticity modulus E_{ur} determines the soil behaviour under unloading and reloading; the indices ur stand for "unloading/reloading". Both the secant virgin loading modulus E_{50} and the unloading modulus E_{ur} are stress-level dependent. It yields:

$$E_{50} = E_{50}^{ref}\left(\frac{c'\cot\varphi' + \sigma_3'}{c'\cot\varphi' + p^{ref}}\right)^m \qquad [2]$$

$$E_{ur} = E_{ur}^{ref}\left(\frac{c'\cot\varphi' + \sigma_3'}{c'\cot\varphi' + p^{ref}}\right)^m \qquad [3]$$

E_{50}^{ref} and E_{ur}^{ref} are input parameters for a particular reference pressure p^{ref}. The exponent m can be measured both in oedometer tests and in triaxial tests. One tends to find values between 0.4 and 1.0. A value of 0.5 is typical for sands and clays tend to have $m \approx 1.0$.

Figure 4 shows the typical curve of an oedometer test. For purposes of comparison with the triaxial curve in Figure 3, the oedometer diagram has been rotated 90° from its normal position, so that the strain axis is horizontal. The virgin oedometer stiffness obeys a stress dependency according to the formula

Fig. 3 Typical curve of a drained triaxial compression test.
Bild 3 *Typischer Kurvenverlauf eines dränierten Triaxialversuchs.*

Fig. 4 Typical curve of a drained oedometer test.
Bild 4 *Typischer Kurvenverlauf eines dränierten Oedometerversuchs.*

$$E_{oed} = E_{oed}^{ref} \left(\frac{c' \cot \varphi' + \sigma_1'}{c' \cot \varphi' + p^{ref}} \right)^m \quad \text{[4]}$$

In the special case of m = 1 one obtains a linear stress-dependency as usual for a clay. In addition to the moduli E_{50}^{ref} and E_{ur}^{ref}, the oedometer modulus E_{oed}^{ref} is also an input parameter. Together with the parameters m, ν_{ur}, c´, φ´ and the dilatancy angle ψ, there are a total of eight material parameters. Within the hardening-softening model c´ is not a constant, but a void ratio dependent parameter as specified by equation 1.

Often, no triaxial test results are available for determining ν_{ur}, E_{ur}^{ref} and E_{50}^{ref}, in which case one has to rely on oedometer results and general empirical data, such as ν_{ur} = 0.1 - 0.2. For sands and stiff clays, one can mostly use $E_{50}^{ref} \approx E_{oed}^{ref}$. However, this equality of reference stiffnesses does not mean that the triaxial stiffness E_{50} equals the oedometer stiffness, E_{oed}. It should be noted that the reference triaxial stiffness is obtained by normalizing to the minor principal stress, σ_3, and the reference oedometer stiffness follows after normalizing to the major principal stress, σ_1

The elasticity modulus E_{ur}^{ref} can be determined directly from a triaxial test or indirectly with the help of oedometer results. If the unloading modulus from the oedometer test is termed E_{oed}^{ur}, according to isotropic linear elasticity the following relationship holds

$$E_{ur} = (1 - 2\nu_{ur}) \frac{1 + \nu_{ur}}{1 - \nu_{ur}} E_{oed}^{ur} \quad \text{[5]}$$

Hence with proper estimates of Poisson's ratio, E_{ur} can be calculated from E_{oed}^{ur}.

Shallow unlined tunnel in softening ground

The authors consider an unlined circular tunnel with a diameter of 8 m and a ground cover of the same thickness. As already mentioned, the authors concentrate on a plane strain situation and analyses can be carried out in a plane strain cross-section. Initial stresses are taken to be geostatic with a coefficient of lateral earth pressure K_0 = 1

Firstly a response curve was computed for a non-cohesive ground to obtain the dashed upper curve in Figure 5 with a failure pressure of $p_f = 0.4\gamma D$. Secondly a non-softening cohesive ground with c′ = 40 kPa was considered to obtain the lower curve in Figure 5 with a slightly negative failure pressure indicating a stable situation. Analyses involving cohesion softening should obviously render ground response curves in between the upper curve for c′ = 0 kPa and the lower curve for c′ = 40 kPa.

In order to model softening in narrow shear

Fig. 5 Computed ground response curves for a shallow tunnel with H/D = 1.

Bild 5 *Berechnete Gebirgskennlinien für einen flachliegenden Tunnel mit H/D = 1.*

bands sufficiently accurate, a very fine mesh around the tunnel were used, as indicated in Figure 6. In fact, such a fine mesh is needed when applying a nonlocal model in combination with a small internal length. First of all a softening analysis was carried out for a stiff clay with a softening modulus h_c = 600 kPa. This yields the blue ground response curve in Figure 5 with a marked peak in point A. Well before peak the blue curve deviates already from the non-softening lower bound and it meets the non-cohesive upper bound finally in point B. The peak point A yields a peak pressure of $p_f = 0.23\gamma D$, being about half way in between the failure pressures for non-softening materials with c′ = 0 kPa and c′ = 40 kPa respectively.

As different clays will have different softening moduli, the softening modulus around the above value of h_c = 600 kPa have been varied . Resulting ground response curves for h_c = 300 kPa and h_c = 1 200 kPa are indicated by the red and green curves in Figure 5. A very slight decrease of the minimum pressure is observed for h_c = 300 kPa and a noticeable increase for h_c = 1 200 kPa.

Considering present computational data for a very shallow tunnel in a particular softening clay, it is thus observed that material softening produces a structural softening in the sense of concave ground response curve as also suggested e.g. by Pacher (10) and Vavrovsky (15).

Figure 6 shows a close up around the tunnel with softening zones at and beyond peak, i.e. for point A, B and C of the blue curve in Figure 5. The yellow zones indicate regions where cohesion has softened down to about 10 kPa. These regions are surrounded by green zones with c´ ≈ 20 kPa. States B and C in Figure 6 show post-peak softening zones with a shear band starting at the tunnel side and gradually growing towards the surface. Here the colour red is used to indicate fully softened soil. Similar results have been obtained by Schuller and Schweiger (12) using a multilaminate model that includes softening behaviour.

Deeper unlined tunnel in softening ground

In this section a deeper tunnel with a cover of H = 32 m is considered and all other parameters conform to the shallow tunnel of the previous section. The initial supporting pressure is given by $p_0 = \gamma (H + 0.5D)$ and this pressure is stepwise reduced to failure. As in the previous section, upper and lower bounds to the ground response curve are obtained for non-softening material with c′ = 0 and c′ = 40 kPa respectively. Instead of showing the full curves starting at $p = 4.5\gamma D$, Figure 7 focuses on the lower part from $p = 2\gamma D$ down to failure. As in the previous section the upper curve reaches a failure pressure of $p_f = 0.4\gamma D$ and the lower curve reaches a slightly

Fig. 6 Development of softening zones for materials with h_c = 600 kPa. Red indicates fully softened material with c´ = 0. First for failure state, then for intermediate state and finally for residual state.

Bild 6 *Entwicklung der Entfestigungszonen für ein Material mit h_c = 600 kPa. Rot bedeutet vollständig entfestigtes Material mit c´ = 0, zunächst im Bruchzustand, dann in einem Zwischenzustand und letztendlich vollständig entfestigt.*

Fig. 7 Computed ground response curves for a deeper tunnel with *H/D* = 4.

Bild 7 *Berechnete Gebirgskennlinien für einen tiefer liegenden Tunnel mit H/D = 4.*

negative failure pressure.

The computed ground response curve for the stiff clay with a softening modulus of h_c = 600 kPa is found to be well in between the bound solutions. However, there is a distinct difference to the response curve of a shallow tunnel. Instead of following the lower bound for $c' = 40$ kPa, the deep-tunnel response curve tends to remain closer to the upper bound for $c' = 0$ kPa. Accordingly the computed peak of $p_f = 0.33\gamma D$ at point A is only slightly below the upper bound of $p_f = 0.4\gamma D$. In fact, there is a difference of only $\Delta p_f = 0.07\gamma D$ with non-cohesive material. At the same time the deeper tunnel is subject to much larger deformations than the shallow tunnel, as can be observed by comparing Figures 5 and 7. No doubt, the relatively large deformations in deep tunnelling induce a relatively large amount of cohesion softening.

Figure 8 shows the development of the softening zone around the tunnel. In the following the authors concentrate on the fully softened red zone. At failure (state A) one observes already a thin red zone and post peak this zone increases rapidly. For state C, one observes the initiation of a shear band towards the surface. For state D this shear band has extended to the surface, but the red part of the band has not yet reached the surface.

Fig. 8 Development of softening zones for a deep tunnel.
Bild 8 *Entwicklung der Entfestigungszonen für einen tiefliegenden Tunnel.*

Conclusions

Attention has been focused on tunnels in softening ground. To study consequences of cohesion degradation, ground response curves have been computed both for a shallow tunnel and a deeper one. The computed ground response curves appear to depend significantly on tunnel depth. For a very shallow tunnel, a trough-like Fenner-Pacher curve is computed with a marked minimum as failure pressure. The deeper the tunnel, however, the smaller the softening behaviour on the structural level of the tunnel. The present study suggests that ground response curves for very deep tunnels will show no softening at all. This is conform to recent numerical studies by Bliem and Fellin (2). Moreover it confirms practical experiences by Vavrovsky (15).

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